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A CLASS OF COMPOSITE LOADS FOR AN INELASTIC MATERIAL

A. F. Revuzhenko UDC 539.37

In solid mechanics a considerable role is played by flow, which in a certain sense is the simplest form. In hydrodynamics this relates to Couette flow between parallel plates and coaxial cylinders [1], in solid mechanics it relates to deformation of thin-walled tubular specimens [2], and in the mechanics of loose materials it relates to uniform shear of the material [3]. Construction of sufficiently general phenomenological models assumes an experimental study of different loading paths, including composite loading paths when the stress tensor axes are turned relative to the volume of the material. Composite loading of metals, rocks, and other solids may be realized by a combination of internal pressure, torsion, and tension for tubular specimens. However, for a broad class of materials this classic procedure is either markedly complicated (e.g., for soils [4]), or it is generally inapplicable. It is of interest to find a class of composite loads which on one hand might relate to the simplest, and on the other might be used in order to test loose, viscoelastoplastic, and other similar materials.

1. As is well known, a uniform stress–strained state is the simplest. Let a material in the fixed direction be subjected to uniform tensile deformation $\Delta\varepsilon_1 = k\Delta t$, and in the orthogonal direction to compressive deformation so that the volume is unchanged; $\Delta\varepsilon_2 = -k\Delta t$. Then after time Δt the same uniform deformation is accomplished in new fixed directions turned relative to the previous directions by angle $-\Omega\Delta t$, etc. Deformation is planar, Ω and k are positive constants.

In order to derive equations, we consider a discrete sequence of these uniform loadings. Let $Ox_1'x_2'$ be the initial Cartesian coordinate system, and β the angle between the tensile direction Ox_1 and axis Ox_1' (Fig. 1). On coordinate Ox_1x_2 the vector for increment

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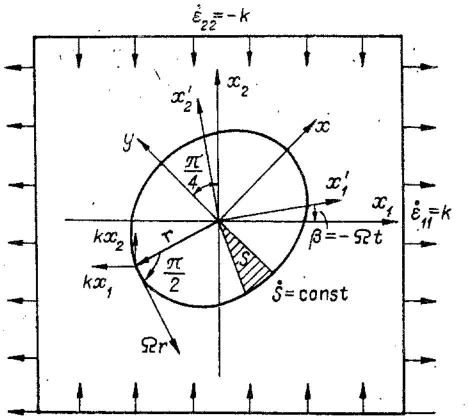

Fig. 1

of displacements is $\{kx_1\Delta t, -kx_2\Delta t\}$. By projecting it onto axes Ox_1' and Ox_2', and changing over to variables x_1' and x_2', we obtain

$$\Delta u_1' = k(\cos 2\beta x_1' + \sin 2\beta x_2')\,\Delta t, \quad \Delta u_2' = k(\sin 2\beta x_1' - \cos 2\beta x_2')\,\Delta t. \tag{1.1}$$

We break down the time intervals from t to t_1 into intervals of length Δt, we assume discrete value $\beta = -\Omega t$, and we sum displacements (1.1) with $\Delta t \to 0$, $t_1 \to t$. As a result of this we have a system

$$v_1' = dx_1'/dt = k(\cos 2\Omega t x_1' - \sin 2\Omega t x_2'), \quad v_2' = dx_2'/dt = -k(\sin 2\Omega t x_1' + \cos 2\Omega t x_2'), \tag{1.2}$$

where v_1' and v_2' are velocities of the material point on coordinates $Ox_1'x_2'$. Now it may be assumed that the system Ox_1x_2 rotates continuously with angular velocity $-\Omega$ relative to the original system. Equations (1.2) make it possible to determine the velocity at coordinates Ox_1x_2:

$$v_1 = dx_1/dt = -\Omega x_2 + kx_1; \quad v_2 = dx_2/dt = \Omega x_1 - kx_2. \tag{1.3}$$

The nature of solution (1.3) depends on the ratio of tensile and rotational velocities. With $k < \Omega$

$$x_1 = \left(\frac{k}{\lambda} a_1 - \frac{\Omega}{\lambda} a_2\right)\sin\lambda t + a_1\cos\lambda t, \quad x_2 = \left(\frac{\Omega}{\lambda} a_1 - \frac{k}{\lambda} a_2\right)\sin\lambda t + a_2\cos\lambda t. \tag{1.4}$$

Here $\lambda = \sqrt{\Omega^2 - k^2}$; a_1 and a_2 are coordinates at instant t_0. If $k \geq \Omega$ then the paths are not closed and they merge at infinity. In this situation the possibility of this behavior at first glance appears paradoxical. However, it has a simple mechanical sense. On polar coordinates (r, α) system (1.3) is transformed to

$$d\ln r/dt = k\cos 2\alpha, \quad d\alpha/dt = \Omega - k\sin 2\alpha.$$

This last equation indicates that the angular velocity of rotation for a material point around the center depends not only on the rotational velocity Ω, but also on tensile velocity k. With $k \geq \Omega$ the radius is found at which the rotational velocity is zero. The point cannot surmount this radius, and as a result of continuous tension it emerges at infinity. Below we limit ourselves only to the first case when $k < \Omega$. In this way according to (1.4) each point moves through a closed elliptical path having a compression coefficient $\sqrt{(\Omega - k)/(\Omega + k)}$. The major axis of the ellipse is directed along bisectrix $x_1 = x_2$. The period of rotation for all of the points is the same and equals $2\pi/\lambda$; i.e., it is always greater than $2\pi/\Omega$. The rule for rotation exhibits one feature: the vector derivative of velocity v at

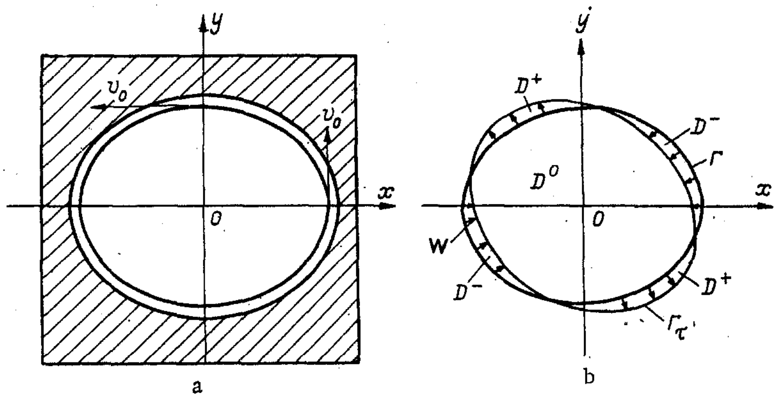

Fig. 2

radius-vector r for all of the points in the overall elliptical path is constant and independent of k. In other words, with movement around the center the radius-vector of the point for the same time covers the same areas S. For example, for points $a_1 = 1$ and $a_2 = 0$ with all t

$$|\mathbf{v}\times\mathbf{r}| = \Omega = \text{const}, \ (\mathbf{v}\cdot\mathbf{n}) = 0, \tag{1.5}$$

where n is normal to the path.

Results (1.4) and (1.5) were obtained by proceeding only from the requirement for uniformity of solid deformation. Loading is quasistatic, and inertia forces are absent. The reason for movement of material elements is their reaction, which leads to development of internal stresses and transmission of information about boundary displacements within the region. The quasistatic problem for flow of a solid may be stated in accordance with the dynamic problem for movement of a material point. Let at a certain instant of time all of the bonds break down between the material elements and instead of them an equivalent force field develops which for any individual particle now provides the same movement as in the solid. We determine the equivalent field for flow (1.3). From (1.3) it follows that

$$d^2x_1/dt^2 = -(\Omega^2 - k^2)x_1, \ d^2x_2/dt^2 = -(\Omega^2 - k^2)x_2.$$

Thus, a central force field with a potential proportional to r^2 corresponds to simultaneous deformation with continuous rotation of the axes. In this language property (1.5) is the well-known consequence of a central force field [6].

2. The simultaneous process is characterized by conditions (1.5). How to realize similar loading? Since material points move through a closed elliptical path, for its realization it is necessary to separate the elliptical region, and at the boundary to prescribe a velocity vector directed along a tangent to the boundary. The value of the velocity should change by rule (1.5). Technically it is quite complicated to accomplish this. It is simpler to retain only the basic features of the simplest uniform situation (transformation of the elliptical region into itself), and to prescribe the linear velocity as constant. This process may be realized as follows. We place the material specimen in a right elliptical cylinder bounded by a flexible sheath. The loading device is made in the form of a rigid vertical cylinder within whose internal plane the flexible sheath with the specimen is placed. Loading is carried out by relative rotation of the outer cylinder and the sheath (Fig. 2a). This method may be generalized in a wider class of regions: a figure of constant diameter, different ovals, etc.

Substitution of boundary conditions (1.5) by

$$|\mathbf{v}| = v_0 = \text{const}, \ (\mathbf{v}\cdot\mathbf{n}) = 0 \tag{2.1}$$

generates a number of questions. First, how informally to present deformation character (2.1)? For this we use the following example. Let Γ and Γ_τ be configuration of the boundary at instants t and t + τ, and D and D_τ be the corresponding regions. Since at the boundary displacements (velocities) are prescribed, then Γ and Γ_τ are known. We combine both configurations. In the general case with superimposition three types of region develop: $D^0 = D \cap D_\tau$, $D^- = D \setminus D^0$, and $D^+ = D_\tau \setminus D_0$. Roughly speaking, the resulting deformation from t to t + τ is reduced to the situation that from D a region of nonconformity D^- is removed and a region D^+ is added. Therefore, their position and form makes it possible before solution of the problem to determine qualitatively the nature of the process as a whole and to give an integral estimate of deformation as the ratio of the areas $D^+ \cup D^-$ to D. Here a situation arises. As a rule, displacements at the boundary are prescribed from expressions of convenience for describing loading-device kinematics. Therefore, these displacements may contain attendant components relating to rigid transfer and rotation of the body being deformed. It is necessary to exclude them. We limit ourselves to problem (2.1) with small times τ. We substitute boundary displacements u_1 and u_2 by $w_1 = u_1 - \Delta\omega x_2$, $w_2 = u_2 + \Delta\omega x_1$. Constant $\Delta\omega$ is determined from the condition

$$\frac{1}{L}\oint_{\Gamma}\frac{\mathbf{w}\times\mathbf{r}}{r^2}\,dl = 0,$$

where L is boundary length. Displacements w_1 and w_2 may be used in order to plot the region of nonconformity for D^+, D^-. It can be seen (Fig. 2b) that their location is such that on the whole the specimen is extended along the direction x = −y and compressed along the orthogonal direction x = y (cf. Fig. 1).

In a more detailed study it is more convenient to proceed from boundary conditions (2.1) although they contain implicitly rigid rotation. This is connected with the fact that in (2.1) the boundary does not change and velocities at it do not depend on time. Therefore, for a broad class of materials the field of velocities and stresses within the region emerges into a steady regime. Apart from direct experimental verification for steadiness it is possible to use the following criterion. We subject some material samples to periodic loading for deformation. If after a certain number of cycles stressed in the sample cease to depend on its original condition (i.e., memory about the original form wears away) and a dependence is only retained on the phase within the cycle, then the material may be referred to the class indicated above.

Let flow (2.1) be steady. As a result of symmetry a material element at the center does not experience transfer, and this means also expansion. Principal directions of the tensor for deformation velocities, velocity of tension and compression along the principal axes, and also rotational velocity will be unknown. This means that the central element is under the "ideal" conditions described above for composite loading with continuous rotation of the axes.

The arrangement of model and rheometric experiments assumes recording of actual data for stress tensors, deformations, and their velocities. In studying composite loading the main question is about coaxiality or the degree of difference for tensors. First we consider a method for determining the deformation tensor component. In this situation there is no basis for assuming rotation and deformation to be small. In order to describe large deformations different measures, as is well known, are used connected with analyzing the change in distance between pairs of close points [7]. Another approach is also possible when attention is concentrated not on relative displacements of points but on transforming some small regions as a whole without "resolving" into displacements for individual points belonging to this region.

Let a_i and x_i be Lagrangian and Euler coordinates of the point, and u_i be displacement vector components:

$$x_i = a_i + u_i(a_j, t),\ i,\ j = 1,2. \tag{2.2}$$

We fix with t = 0 all of the material points within a circle with radius ε and center $a_i = a_i^\circ$. The value of ε may be assumed to be small, but we shall not impose any such limitations on derivatives $\partial u_i/\partial a_j = u_{i,j}$. During deformation the circle changes into an ellipse:

$$(1 + 2E_{22})\,y_1^2 - 4E_{12}y_1y_2 + (1 + 2E_{11})\,y_2^2 = \delta^2, \tag{2.3}$$

where

$$y_i = [x_i - a_i^0 - u_i(a_j^0, t)]/\varepsilon;\ E_{11} = u_{1,1} + (u_{1,1}^2 + u_{1,2}^2)/2;\ E_{22} = u_{2,2} + (u_{2,1}^2 + u_{2,2}^2)/2;\ 2E_{12} = u_{1,2} + u_{2,1} + u_{1,1}u_{2,1} + u_{1,2}u_{2,2};$$
$$\delta = 1 + u_{1,1} + u_{2,2} + u_{1,1}u_{2,2} - u_{1,2}u_{2,1}.$$

It is easy to demonstrate that δ is invariant, and E_{ij} are second-rank tensor components of E. The tensor for deformations E actually coincides with the Finger tensor [8], and it is distinguished from the Green tensor with substitution of derivatives $\partial u_i/\partial a_j$ by $\partial u_j/\partial a_i$. The mechanical meaning of invariant δ and component E is determined by equality (2.3). The principal directions of the tensor coincide with axes of an ellipse into which the surroundings of the point are transformed representing a ring:

$$\mathrm{tg}\, 2\alpha = 2E_{12}/(E_{11} - E_{22}). \tag{2.4}$$

Here α is the angle between the principal directions and the Ox_1 axis.

For the second limiting case, when as a "vicinity" section $a_1 = a_1^0 + \rho\cos\beta_0$, $a_2 = a_2^0 + \rho\sin\beta_0$, $|\rho| < \varepsilon$ is taken, relationship (2.2) leads to an equation for the angle of rotation

$$\mathrm{tg}(\beta - \beta_0) = \frac{u_{2,1} + (u_{2,2} - u_{1,1})\,\mathrm{tg}\,\beta_0 - u_{1,2}\,\mathrm{tg}^2\beta_0}{(1 + u_{1,1}) + (u_{1,2} + u_{2,1})\,\mathrm{tg}\,\beta_0 + (1 + u_{2,2})\,\mathrm{tg}^2\beta_0}.$$

Tensor E makes it possible to follow directly the nature of material deformation during loading of a body. For example, for flow (1.4), $\tan 2\alpha = (\Omega/\lambda)\tan\lambda t$, $\delta \equiv 1$, and the semiaxis of the ellipse referred to the radius of the original circle is $1 \pm (k/\Omega)\sin\Omega t$. Here, in order to shorten the record, it is assumed that $k \ll \Omega$.

From determination of (2.3) a method follows for experimental measurement of component E_{ij}. We mark in the initial instant of time all of the points within a quite small circle. We shall fix parameters of the ellipse into which the circle is transformed during loading of the body. Equation (2.3) makes it possible from these data to set components E_{ij}. Above, tensor E was introduced in view of the fact that measurements of the ellipse parameters is carried out more simply than relative displacements of adjacent points. For the central element measurements confirm fulfillment of equality (1.4), and in addition they make it possible to determine the rotational velocity, magnitude, and principal direction of the tensor for deformation velocity $\dot{\varepsilon}_1$ in flow (2.1).

We move onto the question of stresses. We place at the center of the specimen floating sensors for normal and tangential stresses. The sensors measured stresses between one and the same material particles. In a steady regime corresponding diagrams are periodic over time. Sensor orientation, which corresponds to the extreme of normals or a zero value of tangential components, determines the principal direction of stress tensor σ_1. From the ratio of directions for σ_1 and $\dot{\varepsilon}_1$ it is possible to assess the degree of tensor coaxiality.

Thus, loading scheme (2.1) makes it possible to substitute completely homogeneous scheme (1.5). In addition, an effect is detected which is of independent interest. It is assumed that between the sheath and the material attachment is provided (this condition is not the principal one). According to (2.1) after time L/v_0 all of the boundary points complete an entire revolution and they return to their original position. It is evident that in an elastic specimen all of the internal points also return to their original position. For a broad class of inelastic materials this is not so; in one cycle internal points describe almost closed paths, but they do not return to the original position. This leads to the situation that with an increase in the number of cycles "residual" displacements accumulate. From the point of view of an observer connected with particles at the boundary, the process appears as directional transfer of material elements within a region (Fig. 3, dry sand; in the original condition, half of the specimen was colored black). The effect of directional transfer was observed for viscous liquids, loose, plastic, and a series of other materials exhibiting more complex rheology. The main features of this process may be followed in a model of a Newtonian viscous liquid. The problem is reduced to solution of stationary Navier–Stokes equations

$$\nu\Delta u - \frac{1}{\rho}\frac{\partial p}{\partial x} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}, \tag{2.5}$$
$$\nu\Delta v - \frac{1}{\rho}\frac{\partial p}{\partial y} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}, \quad \frac{du}{dx} + \frac{dv}{dy} = 0,$$

within a region $x^2/(1 + m)^2 + y^2/(1 - m)^2 \le 1$ on condition that at the boundary both velocity components $\mathbf{v} = \{u, v\}$ are prescribed satisfying equality (2.1), where $v_0 = 1$ (see Fig. 2a). Here standard notations are used: x and y are Cartesian coordinates; ν is viscosity;

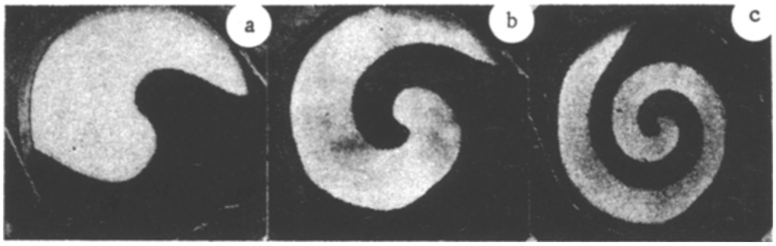

Fig. 3

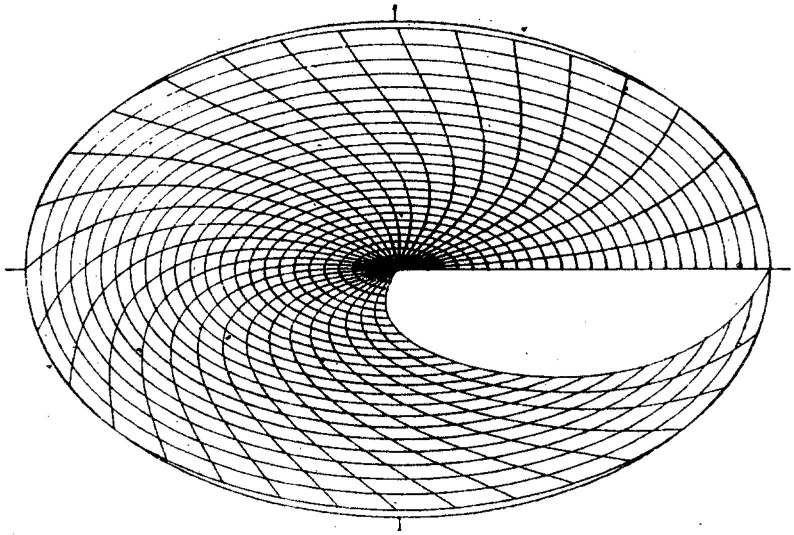

Fig. 4

ρ and p are density and pressure; Δ is Laplace operator. We confine ourselves to the case of high viscosity (Reynolds number Re ≪ 1) and small eccentricities (m ≪ 1). The system is reduced by the small parameter method to a sequence of biharmonic equations relating to terms of resolution for flow functions. By using the scheme in [9], we obtain

$$u = -y + m(-3y + 2y^3) + m^2\left(\frac{7}{4}y + \frac{7}{2}y^3 - \frac{9}{4}y^5 - \frac{21}{2}x^2y + \frac{15}{2}x^2y^3 + \frac{15}{4}x^4y\right) + m^3\left(\frac{87}{8}y - \frac{57}{4}y^3 - \frac{33}{8}y^5 + \frac{5}{2}y^7 + \frac{165}{4}x^2y^3 - \frac{165}{8}x^4y - \frac{105}{4}x^2y^5 + \frac{35}{4}x^6y\right) + \frac{\mathrm{Re}\, m}{16}(x - 2x^3 + x^5 - 6xy^2 + 5xy^4 + 6x^3y^2), \tag{2.6}$$

$$v = x + m(-3x + 2x^3) + m^2\left(-\frac{7}{4}x - \frac{7}{2}x^3 + \frac{9}{4}x^5 + \frac{21}{2}xy^2 - \frac{15}{2}x^3y^2 - \frac{15}{4}xy^4\right) + m^3\left(\frac{87}{8}x - \frac{57}{4}x^3 - \frac{33}{8}x^5 + \frac{5}{2}x^7 + \frac{165}{4}x^3y^2 - \frac{165}{8}xy^4 - \frac{105}{4}x^5y^2 + \frac{35}{4}xy^6\right) + \frac{\mathrm{Re}\, m}{16}(-y + 2y^3 - y^5 + 6x^2y - 5x^4y - 6x^2y^3).$$

Particle transfer and the nature of material deformation were determined by numerical integration* of a set of normal differential equations

$$dx/dt = u(x, y), \quad dy/dt = v(x, y). \tag{2.7}$$

*Experiments and numerical calculations were carried out together with A. P. Bobryakov and V. I. Kramarenko.

Calculations showed that particles move around a center along a closed path. However, the period of rotation in different paths varies. Shown in Fig. 4 are the paths and positions of particles located at first on the major axis of an ellipse (m = 0.2). A difference in periods leads to the situation that internal deformation with increasing time grows without limit. This increase occurs under conditions when external deformations are small (of the order of m). For example, regions initially close to a semicircle during deformation take on a spiral shape (see Figs. 3 and 4). With an increase in the number of cycles they twist more around the center and at the limit they degenerate into two infinitely thin and long spirals which are placed one to the other so that by successively alternating they fill as a whole the original two-dimensional region.

For viscous liquids the role of small parameters m and Re is different in development of flow; parameter m enters into series (2.6) without coefficient Re, but Re only figures in derivatives with m. Therefore, the role of the latter in forming "residual" displacements on a background of parameter m is insignificant. In addition, if we move over to the limit $Re \to 0$ ($\nu \to \infty$), then flow kinematics compared with variant $Re \ll 1$ are almost unchanged, even quantitatively.

Furthermore, the limiting solution Re = 0 is the actual solution of the following linear elastic problem: displacement at the boundary of an elliptical region equals $v_0\Delta t$ and is directed through a tangent to the boundary, but displacement within the region is $u(x, y)\Delta t$, $v(x, y)\Delta t$, where Δt is as small a parameter as you like. We take a new step for the loading parameter, i.e., we prescribe anew increments of boundary displacements $v_0\Delta t$ directed along the boundaries. Since the outline of the boundary after the first step is unchanged, and in addition all of the deformations and rotations are small (of the order of Δt), then for a new step the temptation arises to use the previous solution. Then the resulting displacement of the point (a_1, a_2) should equal

$$\{u(a_i)\Delta t + u(a_1 + u(a_i)\Delta t,\ a_2 + v(a_i)\Delta t)\Delta t,$$
$$v(a_i)\Delta t + v(a_1 + u(a_i)\Delta t,\ a_2 + v(a_i)\Delta t)\Delta t\}$$

etc. By summing a sufficient number of steps from t = 0 to t and directing Δt toward zero, we find that the field of elastic displacements should be determined by integrating systems (2.6) and (2.7) with $Re \equiv 0$ and initial conditions $u \equiv v \equiv 0$. However, integration leads to a situation that with $t = L/v_0$ internal points of the body in the original condition do not return. The result obtained may be considered as an example indicating that solution of the geometrically nonlinear problem cannot be reduced to summing stepwise linear solutions even under conditions when the boundary of the region is always unchanged, and steps for the loading parameter, and also the corresponding deformations and rotations, are exceedingly small. In the general case this question was studied in [10].

On the other hand, correct statement and numerical realization of the stepwise solution should lead to the situation that "residual" displacements for a whole cycle will equal zero. This fact may be used as a test for checking equations, algorithms, and numerical solution programs for elastic geometrically nonlinear problems. This test relates to an essentially two-dimensional arrangement, and large rotations and deformations (if parameter m is not small). In the problem for an elliptical region with boundary conditions (1.5) considerable information is known; apart from absence of "residual" displacements the distribution of strains and stresses should be homogeneous (excluding the case when the material is unstable and bifurcation is possible). These facts may also be taken as tests for inelastic arrangements.

Thus, the procedure and method considered for realizing composite loads may be used in order to study loose, viscoelastoplastic, and other materials for which the classical procedure of testing thin-walled tubular specimens [11] is inapplicable. Solutions for elliptical regions with boundary conditions (1.5) or (2.1) are tests for checking numerical algorithms and the statement of geometrically nonlinear problems. For a broad class of inelastic materials an effect of differential rotation or directional transfer is detected which is also of interest for a number of technical applications [12, 13]. The composite loading process described may be interpreted as a model of the earth's deformation under the action of tidal forces. In this case, the effect of differential rotation means the possibility of a global mechanism of transfer of the earth's mass and its liquid core as a result of the motion of tidal waves [14].

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NONLINEAR WAVES IN A MAXWELLIAN MEDIUM

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UDC 534.2+539.374

The study of propagation of nonstationary nonlinear waves in processes of explosive or shock deformation of metals involves substantial mathematical difficulties and requires, as a rule, a large expense in computer time. In many practical applications the waves generated in the metal during explosion and shock can be assumed to be weak in the sense of smallness of the relative variation of the material density in the wave [1]. Therefore it is of substantial interest to develop approximate methods of analyzing nonlinear waves, based on expanding the solutions in a small given parameter.

To solve nonlinear wave problems in hydrodynamics and elasticity theory it is presently common to develop asymptotic multiple scale methods (MSM) [2-5], making it possible to find uniformly suitable approximations to the solution of the original complex system of equations on some large time interval. The necessity of accounting for strength effects in metals upon explosive deformation or shocks with moderate velocities requires the extension of MSM to more complicated systems of equations, describing, for example, the behavior of a Maxwellian medium [6], which is elastic for small strains, and flows for sufficiently large ones. However, the application of MSM to wave problems in such media is not a formal procedure. This is related to the stress dependence of the kinetic characteristics of the medium (for example, the relaxation time of tangential stresses) in the region of the elastoplastic transition. The latter prevents direct expansion of elastoviscous terms, corresponding to the kinetics, in a series in the small parameter ε (characterizing the relative variation of the material density in the wave) from the initial condition.

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